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ON THE DYNAMICS OF ULTRALONG WAVES IN THE TROPOSPHERE AND LOWER STRATOSPHERE

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ABSTRACT

A linear nonstationary quasi-geostrophic model is considered with external and internal heating and forcing. First, a stationary problem is solved. As it is supposed that the nonstationary component of the ultralong wave is mainly generated by the nonlinear interaction of cyclonic waves, initially it is taken to be the same at all atmospheric levels, with the amplitude and phase being equal to the stationary solution at 700 mb. The total ultralong wave (stationary plus nonstationary components) is taken as the initial value of geopotential in integrating the equations for 90 days. Typically for quasi-stationary ultralong waves, fluctuations at 700 mb and specific (cascade), rather large changes of the amplitudes at the upper levels are obtained. Integral characteristics are quoted that strikingly agree with similar values studied from the real data over the periods of sudden stratospheric warmings and with the results of theoretical investigations of this phenomenon.

Continuing simulation of ultralong waves (Kurbatkin 1967, Kurbatkin and Lenskinov 1968) that consist of two components—stationary and nonstationary (Eliasen and Machenhauer 1965, Deland 1965)—we shall try to simulate and thereby explain their characteristic behavior at different tropospheric and stratospheric levels.

Let us take a nonstationary quasi-geostrophic model with external and internal heating and forcing. Zonal flow is assumed to be independent of the meridional coordinate but changing with height. Vorticity and hydrodynamic equations, in the isobaric coordinate system with upper and lower boundary conditions, are expressed as

$$\frac{\partial \nabla^2 \phi}{\partial t}+U(p)\,\frac{\partial \nabla^2\phi}{\partial x}+\beta\,\frac{\partial \phi}{\partial x}-l^2\,\frac{\partial \omega}{\partial p}=0,$$

$$\frac{\partial^2\phi}{\partial t\partial p}+U(p)\,\frac{\partial^2\phi}{\partial x\partial p}-\frac{dU(p)}{dp}\,\frac{\partial \phi}{\partial x}+\sigma(p)\omega=-\frac{R}{c_p}\,\frac{q}{p},$$

$$\omega=0 \text{ at } p=0,$$

and

$$\omega=-\frac{p}{H}\,U\,\frac{\partial \xi}{\partial x}+\frac{p}{gH}\cdot\frac{\partial \phi}{\partial t} \text{ at } p=p_0.$$

$U(p)$ is the velocity of the geostrophic zonal flow, $\phi=gz$ the geopotential, g the acceleration of gravity, z the height, $\omega=dp/dt$, p is pressure, $p_0=1000$ mb, l the Coriolis parameter, $\beta=dl/dy$; the positive direction of the x axis is from west to east, that of the y axis from south to north, $\sigma(p)$ is the parameter static stability, $\nabla^2=(\partial^2/\partial x^2)+(\partial^2/\partial y^2)$, $q(x,y,p)$ is the diabatic heating per unit mass, R the gas constant, c_p the specific heat at constant pressure, $\xi(x,y)$ the height of the mountain, and $H=p_0/g\rho_0$ is the height of the homogeneous atmosphere.

The function $\phi(x,y,p,t)$ is represented as

$$\phi=[a(p,t)\sin kx+b(p,t)\cos kx]\cos \mu y,$$

where $k=2\pi/L$ and $\mu=\pi/2w$. The functions $\xi(x,y)$ and $q(x,y,p)$ are given as follows: $\xi=\xi_2\cos kx\cos \mu y$, $q=[q_1(p)\sin kx+q_2(p)\cos kx]\cos \mu y$. The coefficient $q_1(p)$ defines diabatic heating and cooling with maximum at the earth's surface, $q_2(p)$ the internal heating and cooling with maximum intensity at 750 mb.

The atmosphere was divided in the vertical into 23 levels from $p=1000$ mb through $p=10$ mb. A system of 46 ordinary linear differential equations was obtained for determination of time-dependent Fourier coefficients of the function ϕ. These equations are not presented here.

Numerical values of the functions $U(p)$, $\sigma(p)$, $q_1(p)$, and $q_2(p)$ are taken from Murakami (1967*b*). In the model considered, $U(p)$ is equal to the half-sum of the zonal velocity values at 35° N. and 60° N. latitude from Murakami (1967*b*). The other parameters are $L=30{,}000$ km, $W=6000$ km, $\xi_2=1$ km, $l=10^{-4}$ sec^{-1}, $\beta=1.6\times10^{-11}$ sec^{-1} m^{-1}, and $H=8$ km.

First, we constructed a stationary solution. Figure 1 shows the amplitude $A(p)=(1/g)\sqrt{[a(p)]^2+[b(p)]^2}$ (in meters) and the vertical variation of the phase angle $\delta(p)=\arctan a(p)/b(p)$ (in degrees) of the stationary wave of the function ϕ/g (solid lines). The wave slopes westward with height, its amplitude increasing in the upper troposphere and stratosphere.

Since we assume that the nonstationary component of the ultralong wave is mainly generated by nonlinear interaction of cyclonic waves and that in the middle troposphere the nonstationary component is comparable with the stationary component in amplitude, initially it was taken to be the same at each atmospheric level, with the amplitude and phase being equal to the stationary solution at 700 mb.*

The total ultralong wave (stationary plus nonstationary components) is taken as the initial value of geopotential in integrating equations for 90 days. Figure 1 shows the initial value of geopotential (dashed line).

Figure 2 shows the amplitudes $A(p,t)$ and phase angles $\delta(p,t)$ of the function ϕ/g at 700, 200 mb (dashed lines), 50 and 25 mb, and figure 3 shows the same functions at 1000, 850, 700, 300, 200, 100, and 50 mb from observed values of the spherical harmonic $m=1$, $n=3$ over the 3-mo period from Dec. 1, 1965, to Feb. 28, 1966.

*It appears that Vitek (1965) was the first to use the barotropic model for studying quasi-barotropic nonstationary ultralong waves.

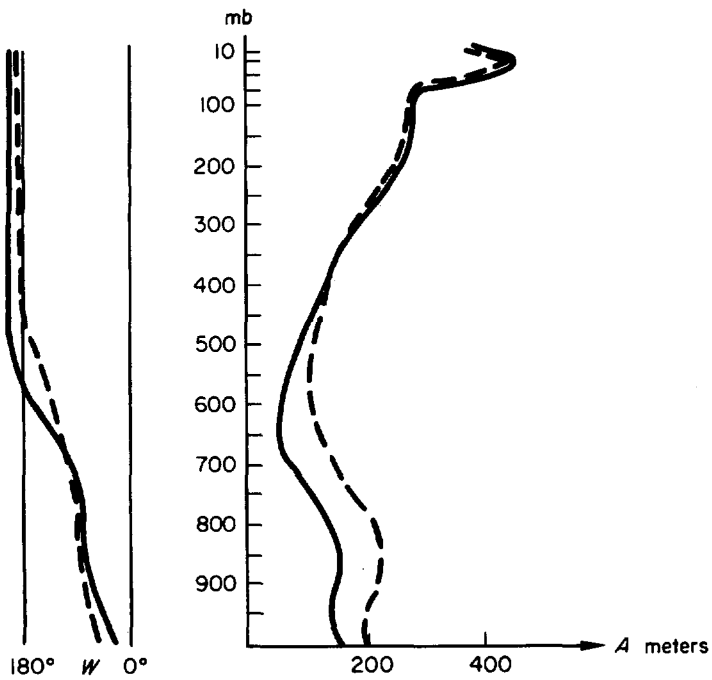

FIGURE 1.—Amplitudes and phase angle of the stationary wave of the function ϕ/g (solid lines) and the initial value of ϕ/g for the time integration (dashed lines). $L=30{,}000$ km, $W=6000$ km, U, σ, q_1, and q_2 are from Murakami (1967b); moreover, we assumed $U(p)=[U(35° \text{ N.})+U(60° \text{ N.})]/2$.

The following feature of the wave behavior deserves attention in figures 2 and 3: there occur, at 700 mb, fluctuations, typical for quasi-stationary ultralong waves, considered earlier in (Kurbatkin 1967) and, at the upper levels, specific (cascade), rather large changes of the amplitudes.

In order to clarify the mechanism of these changes, figure 4 shows separately the time variation of the nonstationary component of our solution. For the same purpose, in figures 2 and 4 the diagrams of the functions $A(t)$ and $\delta(t)$ are superposed for the levels 25 mb (solid lines) and 200 mb (dashed lines).

The cascade decrease of the amplitude of the total ultralong wave, consisting of the stationary and nonstationary components, and its increase afterwards (fig. 2) are stipulated by the change of the vertical inclination of the total wave. This can be clearly seen from the behavior of the phase angles at 25 mb (solid line) and 200 mb (dashed line) at the top of figure 2. Not long after the beginning of the integration, one could observe a negative eastward inclination of the wave with increase of height which, as is well known, prevents the upward energy transport to the stratosphere (the solid line was to the east of the dashed line). Then there occurred a change to positive westward inclination with increase of height. Simultaneously the amplitude increased.

In our model, the sharp sudden increase of the amplitude occurs as a result of the interaction of the stationary and nonstationary components of the ultralong wave during a certain deformation with time of the vertical structure and the movement of the nonstationary component of the ultralong wave. This deformation is shown in figure 4.

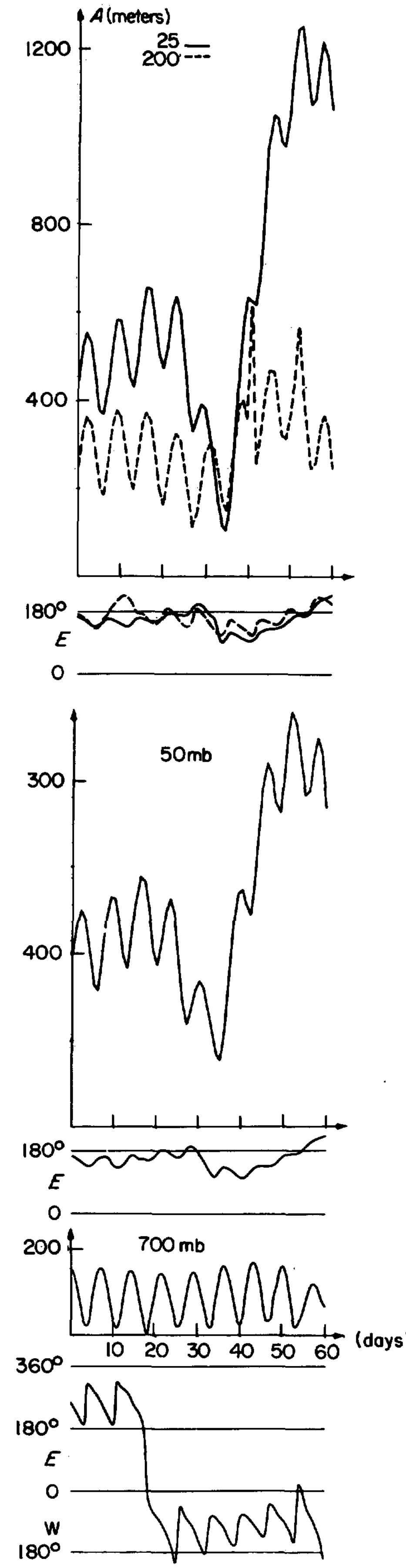

FIGURE 2.—Amplitudes and phase angles of the function ϕ/g for the total ultralong wave (stationary plus nonstationary components) computed from the model.

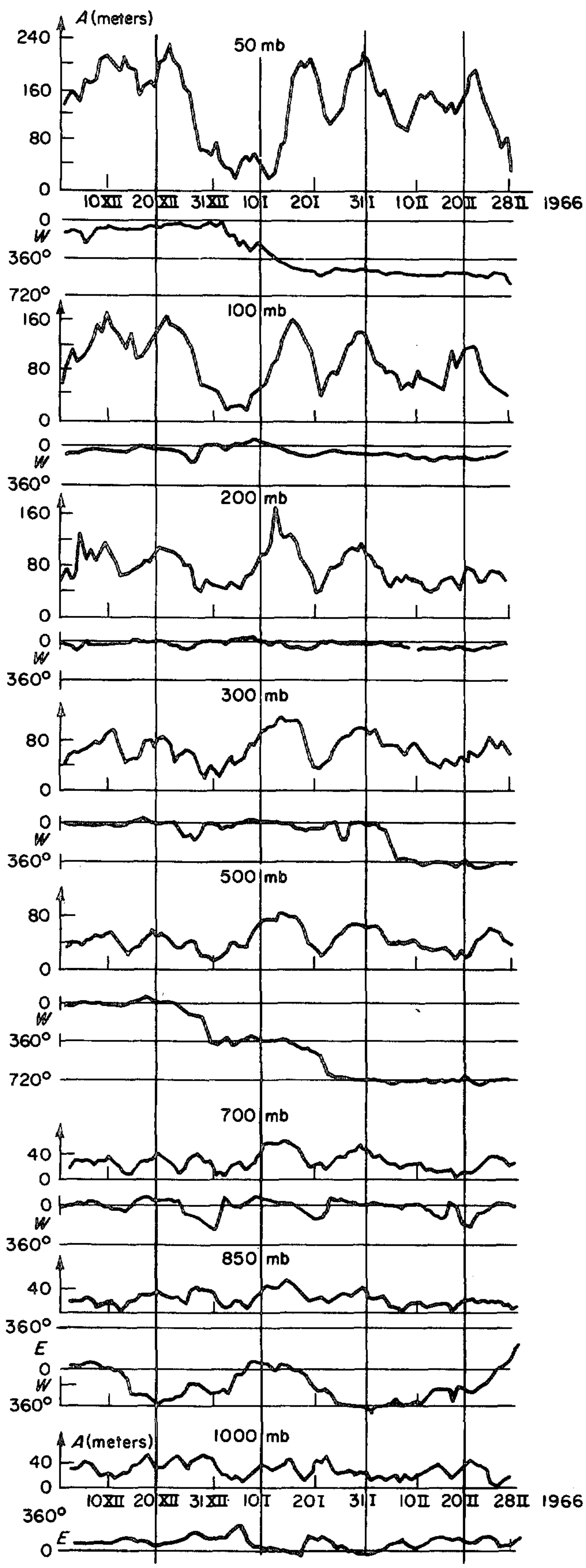

FIGURE 3.—Amplitudes and phase angles of the function ϕ/g according to the observed data for the spherical harmonic $m=1$, $n=3$.

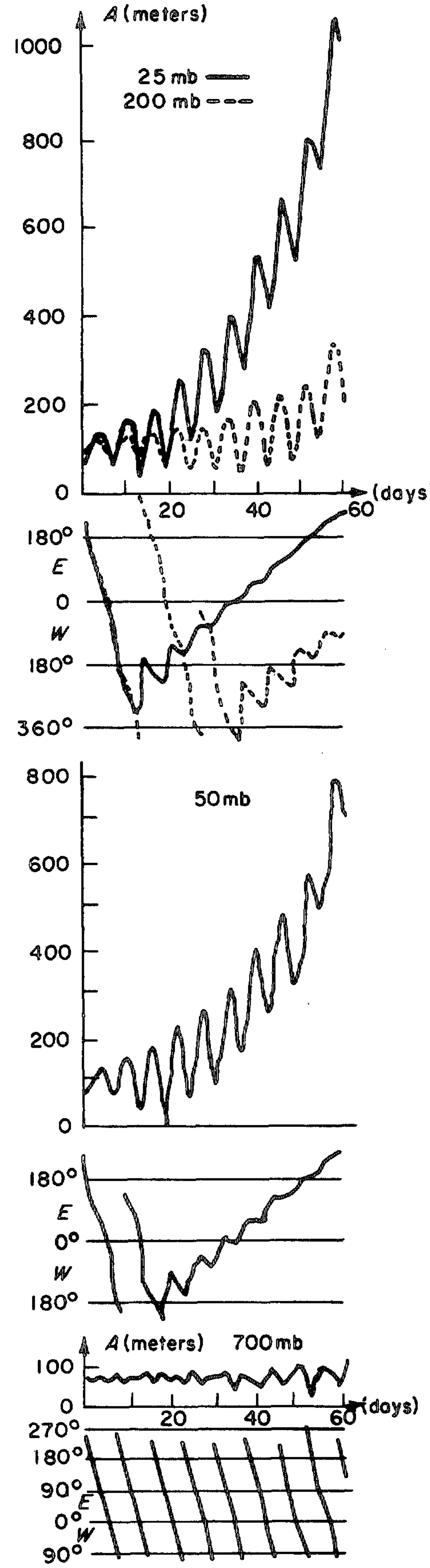

FIGURE 4.—Amplitudes and phase angles of the nonstationary component computed from the model.

Certainly an important part in the process of the sudden increase is played by the external parameters of our model, including $U(p)$ and $\sigma(p)$. They generate a specific form of the stationary solution, described above (fig. 1). The parameters $U(p)$ and $\sigma(p)$ also cause deformation of the nonstationary solutions.

With regard to the initial conditions of our model, it seems to us that, in the real atmosphere, quasi-barotropic nonstationary components of ultralong waves must occur several times in winter during sufficient cyclonic activity as a result of the nonlinear interaction of ultralong waves. At first, such quasi-barotropic nonstationary components will travel westward at all levels, including the upper levels. Then, at some instant there will be a change in the movement of the nonstationary component in the stratosphere: it will begin traveling eastward (as is the case in our model in fig. 4). Sometimes it can result in a sudden increase of the amplitude in the stratosphere, as happened, for example, on Jan. 12, 1966 (fig. 3).

Eastward movements of the nonstationary component of certain ultralong waves in the first half of January 1966, then westward movements, and, after February 10, eastward movements again were observed in the real stratosphere (Hirota 1968*b*).

The deformation of the ultralong wave in the stratosphere, described above, appears to be stipulated by the occurrence and by the intensification, some time later, of the baroclinic and baroclinically unstable modes in the nonstationary solution. The nonstationary ultralong waves in the baroclinic flow were studied in more detail in Hirota (1968*a*).

Calculations were also made for W=4500 km and W=3000 km. In each case, an abrupt increase of the amplitude occurs on the 36th day. The time integration was carried out with the time step eight times diminished. The results showed practically no difference.

Figure 5 shows the heat flows $\overline{vT}=(1/2\pi)\int_0^{2\pi} vT\,dx$, computed from the observed data, across 45° N. latitude for the spherical harmonic $m=1$, $n=3$ over the same 3-mo period of time from Dec. 1, 1965, to Feb. 28, 1966. It appears that in the stratosphere, simultaneously with the sharp increase of the amplitude of the ultralong wave, one observes the transport of large quantities of heat toward the north. One should then expect a warming of the whole polar stratosphere. However, this is a nonlinear process, and it will not be considered in the present paper.

For the total solution, consisting of the stationary and nonstationary components, we have computed the integral values: 1) $(1/g)\int_0^L\int_{10\text{ mb}}^{200\text{ mb}} 10(\partial\phi/\partial p)dp\,dx$ (solid line in fig. 6) and $(1/g)\int_0^L\int_{250\text{ mb}}^{1000\text{ mb}} \omega(\partial\phi/\partial p)dp\,dx$ (dashed line in fig. 6)—measures of transformation of the available potential energy into kinetic energy (if>0); 2) $(1/g)\int_0^L \omega\phi\,dx$—the value of the upward flow of geopotential (if<0), figure 7; 3) $-(p/RL)\int_0^L v(\partial\phi/\partial p)\,dx$—a measure of the northward heat transfer (if>0), figure 8, as well as the amplitude of the temperature wave $|T|$ as a function of p and t, figure 9. These values strikingly agree with similar values obtained from real data over the periods of sudden stratospheric warmings (Scherhag 1960, Miyakoda 1963, Murakami 1967*a*, Sawyer 1965, Deland and Johnson 1968) and with the results of theoretical studies of this phenomenon (Charney and Drazin 1961, Charney and Pedlosky 1963, Nitta 1967, Manabe and Hunt 1968).

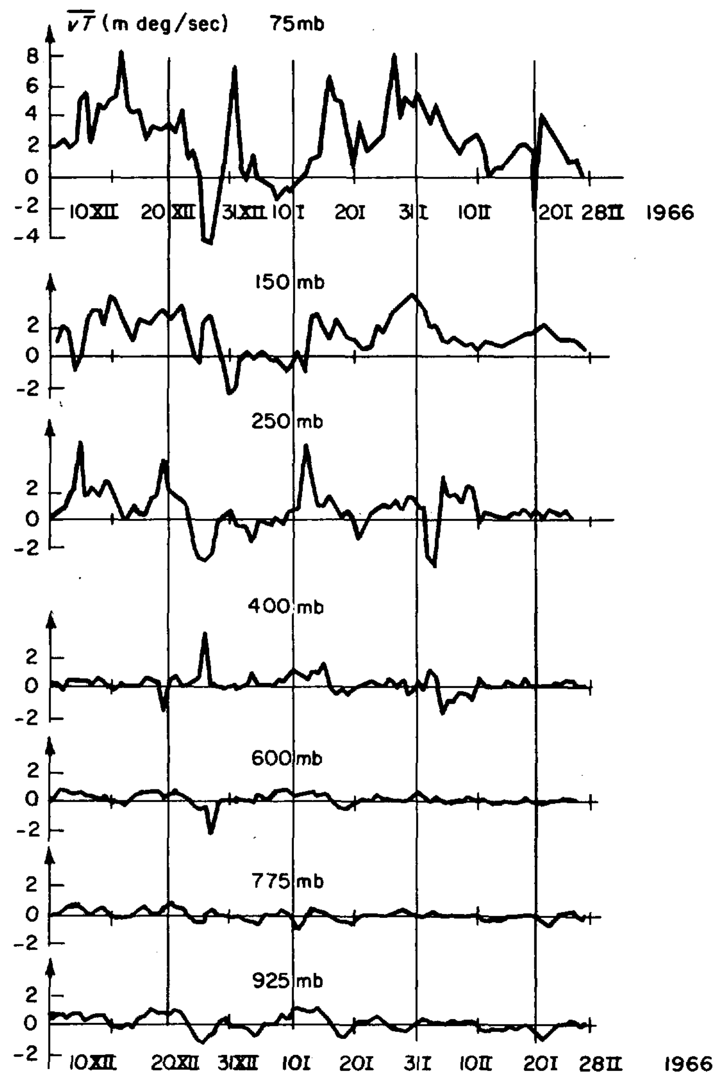

FIGURE 5.—Observed sensible heat transfer, $\overline{vT}=(1/2\pi)\int_0^{2\pi} vT\,d\lambda$, across 45° N. latitude. Northward transfer is positive.

Figure 10 shows amplitudes and phase angles at the levels 200 mb (dashed line) and 25 mb for the case when W=3000 km and $U(p)$ corresponds to the normal winter profile of the mean zonal flow at 60° N. latitude (Murakami 1967*b*) with the polar night jet at 30 mb. The abrupt increase of amplitudes in this example begins on the 57th day.

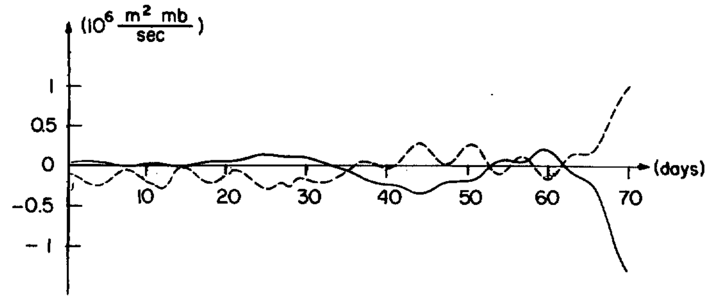

FIGURE 6.—Measure of conversion (positive values) of available energy into kinetic energy $(1/g)\int_0^L\int_{10mb}^{200mb}\omega(\partial\phi/\partial p)dp\,dx$ (solid line) and $(1/g)\int_0^L\int_{250mb}^{1000mb}\omega(\partial\phi/\partial p)dp\,dx$ (dashed line), computed from the model.

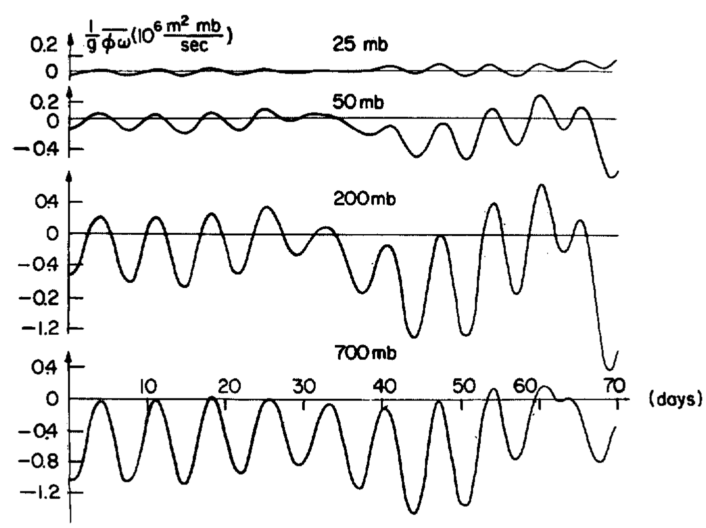

FIGURE 7.—Upward transfer (positive values) of geopotential $(1/g)\int_0^L \omega\phi\, dx$ computed from the model.

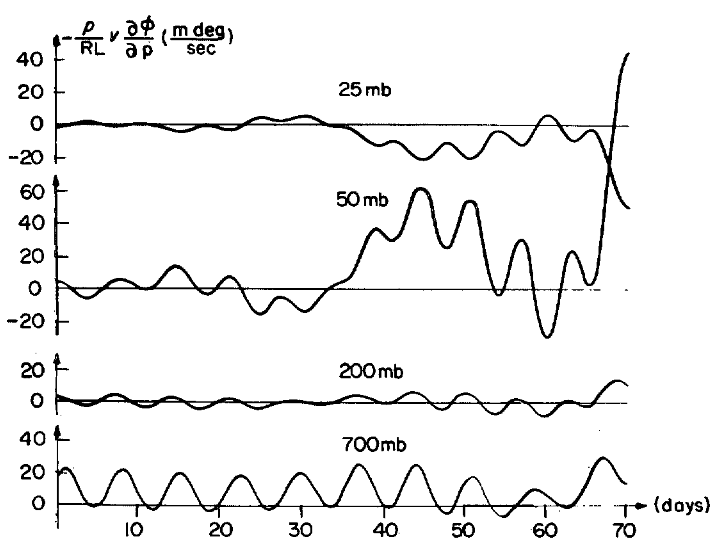

FIGURE 8.—Measure of northward sensible heat transfer (positive values) $(-p/RL)\int_0^L v(\partial\phi/\partial p)dx$ across the parallel $y=0$.

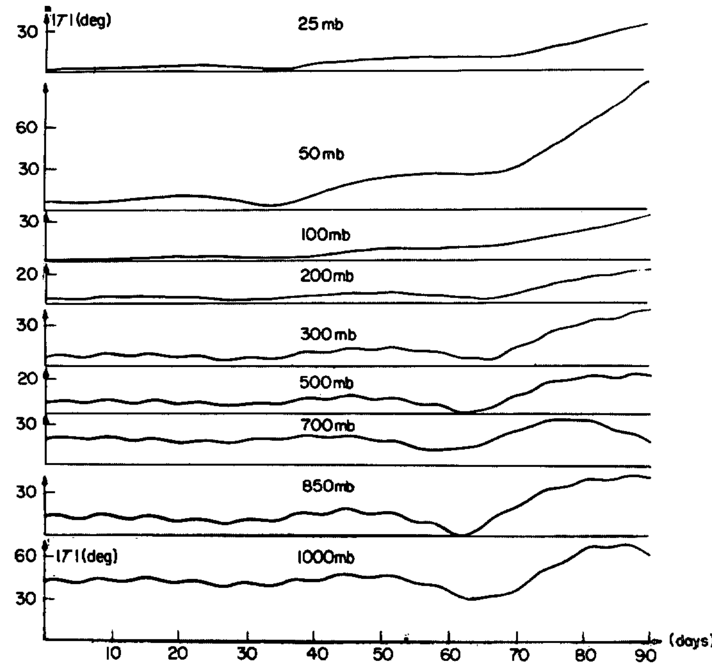

FIGURE 9.—Amplitude of the temperature wave computed from the model.

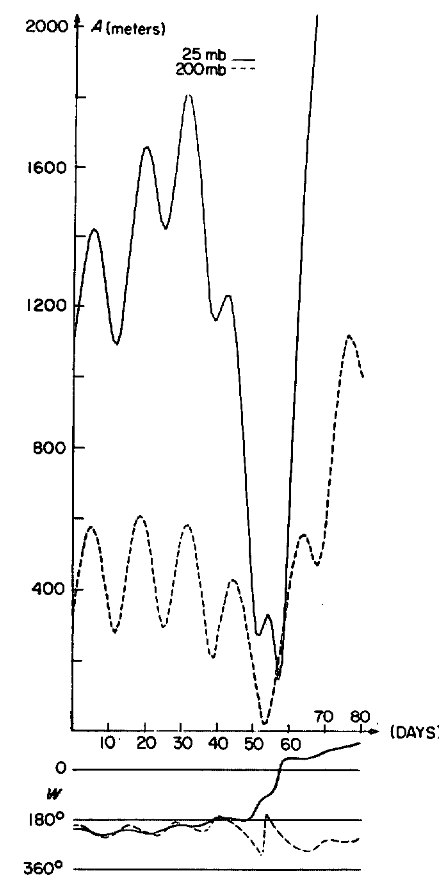

FIGURE 10.—Amplitudes and phase angles of the function ϕ/g computed from the model for the total ultralong wave for values of the parameters $W=3000$ km and $U(p)=U(60°\text{ N.})$ (Murakami 1967); the other parameters are kept the same.

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